CONVECTION IN A MEDIUM WITH SPIRAL TURBULENCE

Yu. A. Berezin and V. P. Zhukov

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The present authors carried out a detailed study of the linear stage of convective instability of coarse scale perturbations in the presence of gyrotropic or spiral turbulence in [1] using the equations of [2]. It was shown that with increase in spirality the horizontal dimension of convective cells increases and when a critical spirality is reached complete readjustment of the flow occurs, with formation of a vortex, the size of which is determined by the horizontal inhomogeneity of the problem.

Now, within the framework of the equations of [1, 2] we will study the nonlinear stage of convective instability. The basic equations have the form

$$\partial \mathbf{u}/\partial t + (\mathbf{u}\nabla)\mathbf{u} = (-1/\rho_0)\nabla p + \nu\Delta\mathbf{u} + \beta g\theta\mathbf{e} + \beta g A\mathbf{f},$$
$$\partial\theta/\partial t + (\mathbf{u}\nabla)\theta = A(\mathbf{eu}) + \chi\Delta\theta,\ \operatorname{div}\mathbf{u} = 0,$$
$$\mathbf{f} = \mathbf{e}(\mathbf{e}\operatorname{rot}\lambda\mathbf{u}) - (\mathbf{e}\nabla)[\mathbf{e}\cdot\lambda\mathbf{u}],\ \mathbf{e} = (0, 0, 1),$$

where $\lambda = \lambda_0\alpha(r, z)$ is the spirality coefficient, λ_0 = const; the remaining notation coincides with that of [1]. In dimensionless variables we have

$$\partial \mathbf{u}/\partial t + (\mathbf{u}\nabla)\mathbf{u} = -\nabla p + \Delta\mathbf{u} + \mathrm{Ra}\,\theta\mathbf{e} + s\mathbf{f},\qquad (1)$$
$$\partial\theta/\partial t + (\mathbf{u}\nabla)\theta = (\mathbf{eu}) + \Delta\theta,\ \operatorname{div}\mathbf{u} = 0,$$
$$\mathbf{f} = \mathbf{e}(\mathbf{e}\operatorname{rot}\alpha\mathbf{u}) - (\mathbf{e}\nabla)[\mathbf{e}\cdot\alpha\mathbf{u}].$$

Here $\mathrm{Ra} = \beta g A H^4/\nu^2$; $s = \mathrm{Ra}\lambda_0\nu/H^3$; H is the height of the liquid layer; H, $t_0 = H^2/\nu$, $u_0 = \nu/H$, $p_0 = \rho_0\nu^2/H^2$, $T_0 = AH$ are length, time, velocity, pressure, and temperature scales.

We will consider the two-dimensional problem for a planar layer in the variables flow function—vorticity

$$\omega = \partial u/\partial z - \partial w/\partial x,\ u = \partial\psi/\partial z,\ w = -\partial\psi/\partial x,$$

having written Eq. (1) in the form

$$\frac{\partial\omega}{\partial t} + \frac{\partial(\psi, \omega)}{\partial(z, x)} = \Delta\omega - \mathrm{Ra}\frac{\partial\theta}{\partial x} + s\left(\frac{\partial^2\alpha v}{\partial z^2} - \frac{\partial^2\alpha v}{\partial x^2}\right),\qquad (2)$$
$$\frac{\partial v}{\partial t} + \frac{\partial(\psi, v)}{\partial(z, x)} = \Delta v - s\frac{\partial\alpha v}{\partial z},\quad \frac{\partial\theta}{\partial t} + \frac{\partial(\psi, \theta)}{\partial(z, x)} = \Delta\theta + w,$$
$$\Delta\psi = \omega,\quad \frac{\partial(\psi, f)}{\partial(z, x)} = \frac{\partial\psi}{\partial z}\frac{\partial f}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial f}{\partial z},\quad \mathbf{u} = (u, v, w).$$

The z axis is directed across the layer and the x axis along it.

We will find the steady state ($\partial/\partial t = 0$) nonlinear solution of system (2) in the limited region $0 \le z \le 1$, $0 \le x \le x_0$ with periodic boundary conditions for both coordinates in the case of slight supercriticality, where the Rayleigh number Ra slightly exceeds the critical number Ra_*. The spirality is considered homogeneous ($\alpha = 1$). The increment of the most rapidly increasing harmonic is small, so it can be expected that the weak nonlinearity stabilizes the growth of the perturbation even at a small amplitude of the latter.

We rewrite the steady state analog of system (2) in the form

$$L_0\mathbf{f} = \partial(\psi, \mathbf{f})/\partial(z, x),\qquad (3)$$

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where

$$\mathbf{f}=(\omega,\, v,\, \theta,\, \psi), \quad L_0\mathbf{f}\equiv-\begin{pmatrix} \mathrm{Ra}\,\partial\theta/\partial x - s\,(\partial^2 v/\partial z^2 - \partial^2 v/\partial x^2) \\ s\partial^2\psi/\partial z^2 \\ \partial\psi/\partial x \\ \omega \end{pmatrix} + \Delta\mathbf{f},$$

and following the usual procedure (see, for example, [3]), write its solution in the form of a series in the small parameter ε, characterizing the degree of liquid supercriticality:

$$\mathbf{f}=\varepsilon\mathbf{f}^{(1)}+\varepsilon^2\mathbf{f}^{(2)}+\ldots, \quad \varepsilon^2=\mathrm{Ra}-\mathrm{Ra}_*.$$

Then Eq. (3) may be written as

$$L\mathbf{f}=\partial(\psi,\,\mathbf{f})/\partial(z,\,x)+(\varepsilon^2\partial\theta/\partial x)\boldsymbol{\eta}, \tag{4}$$

where

$$\boldsymbol{\eta}=(1,\,0,\,0,\,0), \quad L\mathbf{f}\equiv\Delta\mathbf{f}-\begin{pmatrix} \mathrm{Ra}_*\,\partial\theta/\partial x - s\,(\partial^2 v/\partial z^2 - \partial^2 v/\partial x^2) \\ s\partial^2\psi/\partial z^2 \\ \partial\psi/\partial x \\ \omega \end{pmatrix}.$$

Equating terms with equal powers of ε in Eq. (4), we obtain equations of successive approximations

$$L\mathbf{f}^{(1)}=0,\; L\mathbf{f}^{(2)}=\partial(\psi^{(1)},\,\mathbf{f}^{(1)})/\partial(z,\,x); \tag{5}$$

$$L\mathbf{f}^{(3)}=\frac{\partial(\psi^{(1)},\,\mathbf{f}^{(2)})}{\partial(z,\,x)}+\frac{\partial(\psi^{(2)},\,\mathbf{f}^{(1)})}{\partial(z,\,x)}+\frac{\partial\theta^{(1)}}{\partial x}\boldsymbol{\eta}. \tag{6}$$

Using the periodic boundary conditions we obtain from these equations

$$\mathbf{f}^{(1)}=\beta_1\begin{pmatrix} -K^2\sin kx\sin\pi z \\ (s\pi^2/K^2)\sin kx\sin\pi z \\ (k/K)\cos kx\sin\pi z \\ \sin kx\sin\pi z \end{pmatrix}\equiv\beta_1\boldsymbol{\xi}, \quad K^2=\pi^2+k^2; \tag{7}$$

$$\mathbf{f}^{(2)}=\beta_2\boldsymbol{\xi}+\begin{pmatrix} 0 \\ 0 \\ -\beta^2\,(k^2/8\pi K^2)\sin 2\pi z \\ 0 \end{pmatrix} \tag{8}$$

[$k = \pi n/x_0$, n is an integer which is chosen such that $\mathrm{Ra}_* = (K^6 - s^2\pi^2(\pi^2 - k^2))/k^2$, calculated for a given n value is at a minimum; the coefficients β_1 and β_2 are defined by the condition of solubility of the third and fourth approximation equations, respectively].

We now substitute Eqs. (7), (8) in Eq. (6). Then

$$L\mathbf{f}^{(3)}=\begin{pmatrix} \beta_1\,(k/K)^2\sin kx\sin\pi z \\ 0 \\ \pi\beta_1\beta_2\,(k^2/K^2)\sin 2\pi z-\beta_1^3\,(k^3/8K^2)\cos kx\,(\sin\pi z-\sin 3\pi z) \\ 0 \end{pmatrix}. \tag{9}$$

We seek the solution of Eq. (9) in the form

$$\begin{gathered} \omega^{(3)}=(A_1\sin\pi z+A_2\sin 3\pi z)\sin kx, \\ \theta^{(3)}=(B_1\sin\pi z+B_2\sin 3\pi z)\cos kx+B_3\sin 2\pi z,\ldots \end{gathered} \tag{10}$$

The condition of solubility of the system of linear algebraic equations for the amplitudes A_1, A_2, ..., obtained after substitution of Eq. (10) in Eq. (9), allows us to find the coefficient $\beta_1 = \pm(K/k)(8/\mathrm{Ra}_*)^{1/2}$.

Thus, the amplitude of the steady state nonlinear regime is proportional to the quantity

$$\varepsilon\beta_1=(K/k)\,[8\,(\mathrm{Ra}-\mathrm{Ra}_*)/\mathrm{Ra}_*]^{1/2}.$$

We will consider steady state nonlinear solutions for slight supercriticality for the case of an infinite horizontal layer ($0 \le z \le 1$, $-\infty \le x \le \infty$):

$$f = \varepsilon f^{(1)} + \varepsilon^2 f^{(2)} + \ldots,\ \mathrm{Ra} = \mathrm{Ra}^{(0)} + \varepsilon^2 \mathrm{Ra}^{(2)} + \ldots$$

It is well known that we may use as a measure of the intensity of convective motion the thermal flux $\langle w\theta\rangle = \int w\theta dV / \int dV$, which is proportional to the difference between all the heat transported through the layer and the heat which is transported solely due to thermal conductivity for a given equilibrium temperature gradient. We therefore normalize the steady state solution for the case considered to this thermal flux: $\langle w\theta\rangle = \varepsilon^2$, or

$$\langle w^{(1)}\theta^{(1)}\rangle = 1,\ \langle w^{(1)}\theta^{(2)} + w^{(2)}\theta^{(1)}\rangle = 0,\ \ldots \qquad (11)$$

The first approximation solution has the form of Eq. (7) with coefficient $\beta_1 = 2K/k$, obtained by normalization of Eq. (11). The wave number k in the horizontal direction is defined as the value at which the neutral curve reaches a minimum, i.e., $\mathrm{Ra}^{(0)} = \min[(K^6 - s^2\pi^2(\pi^2 - k^2))/k^2]$.

To solve the second approximation equations described by Eq. (8), normalization of Eq. (11) yields $\beta_2 = 0$, so that in the second approximation the correction to the turbulence and flow function (and correspondingly, the velocity) is absent, and in the expressions for temperature we have the second harmonic of the coordinate z, independent of x. In the case under consideration the equations of the third approximation, Eq. (6) has the form

$$L\mathbf{f}^{(3)} = \frac{\partial(\psi^{(1)}, \mathbf{f}^{(2)})}{\partial(z, x)} + \frac{\partial(\psi^{(2)}, \mathbf{f}^{(1)})}{\partial(z, x)} + \mathrm{Ra}^{(2)}\frac{\partial\theta^{(1)}}{\partial x}\eta.$$

Substituting on the right side of the solution of the first and second approximations (7), (8), with coefficients $\beta_1 = 2K/k$, $\beta_2 = 0$ and using the condition of solubility of the inhomogeneous equation thus obtained, which consists of orthogonality of the right side to the eigenfunctions of the operator L, we find $\mathrm{Ra}^{(2)} = (1/2)\mathrm{Ra}^{(0)}$, or $\varepsilon^2 = 2(\mathrm{Ra} - \mathrm{Ra}^{(0)})/\mathrm{Ra}^{(0)}$.

In the third approximation the expressions for velocities and temperature contain the third harmonic of the coordinate z.

Increase in spirality above some limit leads to convective instability, for which instead of a set of cells with approximately equal vertical and horizontal dimensions, formation of a structure with horizontal dimensions significantly greater than the vertical becomes energetically more favorable. When the spirality $s \to s_*$ this dimension formally tends to infinity, and a limited horizontal structure scale can be obtained by introducing horizontal inhomogeneity into the problem, for example, a dependence of spirality on the coordinate x.

We will now consider the results of numerical solution of the general nonlinear problem of convection under conditions of developed spiral turbulence for the axisymmetric case in the region $0 \le z \le 1$, $0 \le r \le r_0$ ("disk"), when the spirality is a function of the radial coordinate: $\alpha(r) = 1 - (\delta r)^2$. System (1) takes on the form

$$\frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial r} + w\frac{\partial\omega}{\partial z} - \frac{u\omega}{r} - \frac{2v}{r}\frac{\partial v}{\partial z} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\omega r}{\partial r}\right) + \frac{\partial^2\omega}{\partial z^2} - \mathrm{Ra}\frac{\partial\theta}{\partial r} + s\left(\alpha\frac{\partial^2 v}{\partial z^2} - \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\alpha vr}{\partial r}\right)\right), \qquad (12)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial vr}{\partial r}\right) + \frac{\partial^2 v}{\partial z^2} - s\alpha\frac{\partial u}{\partial z},$$

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial r} + w\frac{\partial\theta}{\partial z} = w + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) + \frac{\partial^2\theta}{\partial z^2},$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right) + \frac{1}{r}\frac{\partial^2\psi}{\partial z^2} = \omega,\quad u = \frac{1}{r}\frac{\partial\psi}{\partial z},\quad w = -\frac{1}{r}\frac{\partial\psi}{\partial r}.$$

The following boundary conditions were chosen:

$u = v = w = 0$ (or $\psi = \partial\psi/\partial n = v = 0$, where n is the normal to the corresponding boundary) at the boundaries $z = 0$, $z = 1$, $r = r_0$; $\theta = 0$ on the boundaries $z = 0$ and $z = 1$; $\partial\theta/\partial r = 0$ on the boundary $r = r_0$; $\psi = v = \omega = \partial\theta/\partial r = 0$ on the axis $r = 0$. (13)

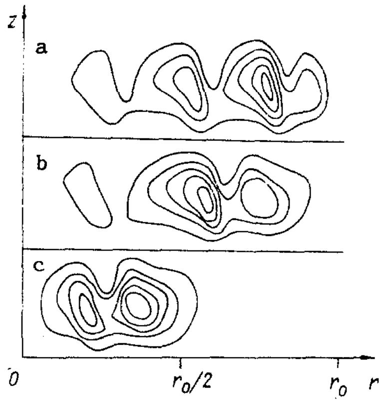

Fig. 1

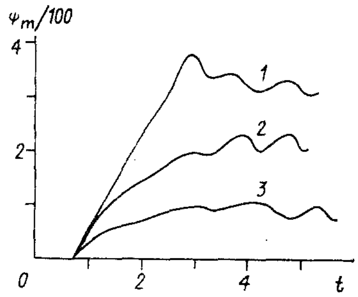

Fig. 2

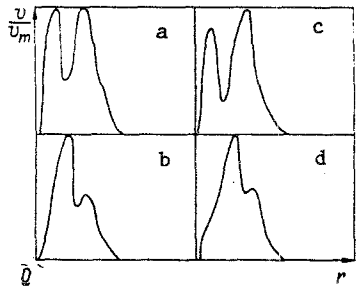

Fig. 3

At time t = 0 initial conditions

$$\omega(r, z) = \exp\left\{-\left(\frac{r-a_1}{a_1}\right)^2 - \left(\frac{z-a_2}{a_2}\right)^2\right\}, \quad (14)$$
$$v(r, z) = \theta(r, z) = 0.$$

were specified.

For the numerical solution of Eqs. (12)-(14) an explicit difference scheme with order of approximation $O(\tau, h_1^2, h_2^2)$ was used, where τ, h_1, h_2 are the steps in t, r, z, respectively. The values of the flow function ψ, turbulence ω, and azimuthal velocity v were calculated at the points $r_i = ih_1$, $z_j = jh_2$, and temperatures at the points $r_{i+1/2} = (i + 1/2) \times h_1$, $z_j = jh_2$. The Poisson equation for the flow function ψ was solved by a single-time rapid Fourier transform with respect to the index j with subsequent drive along the index i.

We will now present the results of numerical solution of Eqs. (12)-(14) for a "disk" with parameters $r_0 = 20$, Ra = 800, s = 8, $a_1 = 3$, $a_2 = 0{,}5$, $\tau = 10^{-3}$, $h_1 = 1/3$, $h_2 = 1/8$. Figure 1 shows flow function isolines for δ = 0; 0.023; 0.046 (a-c), characterizing the degree of horizontal inhomogeneity of the spirality. We use the symbol r_* to denote the radius at which the perturbation increment calculated from the local spirality value $s_\ell(r) = s\alpha(r)$ vanishes, i.e., $\max_k \gamma(\text{Ra}, s_\ell(r), k^2) = 0$, $\gamma|_{r<r_*} > 0$, $\gamma|_{r>r_*} < 0$ (a-c: $r_* = \infty$; r_0; $r_0/2$). Figure 2 shows the quantity $\psi_m = \max_{i,j} \psi_{i,j}$ as a function of time for δ = 0; 0.023; 0.046 (1-3). Figure 3 gives the radial distribution of azimuthal velocity v(r, z) for z = 0.5 and $\delta = 0.046$ ($r_*/r_0 = 0.5$) at various times (a-d: t = 3.5, 4.0, 4.5, 5.0), normalized to the maximum azimuthal velocity in the calculation region $v_m = \max_{i,j} v_{i,j}$ (a-d, v_m = 80, 114, 92, 113).

Analysis of the calculation results permits the following conclusions: 1) in the linear stage (t < 0.5) the unknown functions increase exponentially. Then in the time interval 1.5 < t < 3 the nonlinear terms become significant, and the maximum values of the flow function ψ_m and azimuthal velocity v_m increase linearly: $\psi_m, v_m \sim (t - t_0)$. At t > 3 the quantities ψ_m and v_m undergo oscillations with period $T \approx 1$ about some constant values. These oscillations correlate with the number of extrema in the azimuthal velocity v(r, z = 0.5). The presence of these oscillations does not contradict the first part of this study, which constructed a steady state solution for the case of slight supercriticality, since the values of s and Ra in the given calculation are far from critical; 2) with increase in the spatial inhomogeneity parameter δ the flow localization region becomes smaller, coinciding with the zone in which the linear increment γ, calculated from the local spirality value $s_\ell(r) = s\alpha(r)$, is positive.

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INFLUENCE OF PHASE TRANSITIONS ON SOUND PROPAGATION IN FOGS: COMPARISON OF THEORY WITH EXPERIMENT

D. A. Gubaidullin and A. I. Ivandaev

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Several theoretical and experimental papers [1-10] have been devoted to the propagation of acoustic disturbances in two-component mixtures of a gas with vapor and liquid droplets. Here we give a brief survey of the latest theoretical publications. We discuss the existing experimental data. We also compare the theory developed in [9] with the experimental data of other authors.

1. The work of Cole and others [2, 4, 5] can evidently be cited among the earliest theoretical studies of the propagation of low-intensity waves in two-component two-phase mixtures of an inert gas with a vapor and liquid droplets in the presence of mass transfer by diffusion. These authors investigated the case of small mass contents of the condensed phase, $m \ll 1$. It was established [2, 5] that the first maximum of the attenuation per wavelength σ in aerosols with phase transitions occurs at $\omega\tau_V \sim m$ (ω is the angluar frequency, and τ_V is the Stokes relaxation time of the phase velocities; see Sec. 3 below), i.e., at $\omega\tau_V \ll 1$. The attenuation coefficient in the vicinity of frequencies $\omega\tau_V \sim m$ is much greater than the corresponding values of σ for aerosols without phase transitions. A previous comparison [3] of theory with experiment indicated only qualitative agreement between them.

Marble and Candel [6] investigated the feasibility of using a cloud of fine droplets to attenuate noise with the injection of liquid into the air intake of a turbojet engine. The magnitude of such attenuation is proportional to the vapor concentration k_V in the gaseous phase, but this rule does not hold for large values of k_V. The shortcoming of [2, 4-6] lies in the failure to take into account the difference between the gas constants of the vapor and gas components in the equation of state for the host phase. It was actually assumed, therefore, that the gaseous phase is a calorically ideal gas when mass transfer is present in the disperse system. Allowance for the indicated difference in [7] improved the agreement between theory and the experimental data. However, this agreement still fell short.

All of the cited investigations of sound propagation in vapor–gas–droplet systems were carried out within the framework of a quasiequilibrium phase transition scheme, where it is assumed that the temperature of the droplet surface during mass transfer is equal to the saturation temperature at the given partial pressure of the vapor. The transient effects of phase interaction are significant for high-frequency disturbances in the suspension; when they are taken into account, in general, the effects of nonequilibrium of the phase interface in phase transition are also taken into account. The influence of the sum total of transient and nonequilibrium effects of interphase mass, momentum, and energy transfer on the propagation of acoustic disturbances in mixtures of a gas with vapor and liquid droplets was first investigated by Gubaidullin and Ivandaev [9, 10]. They analyzed the individual contributions of nonequilibrium interphase heat and mass transfer and friction of the phases to wave dispersion and dissipation.

From the experimental point of view, the propagation of weak disturbances in gas suspensions has not been adequately studied to date. The majority of experimental studies have been concerned with sound propagation in suspensions without phase transitions. Accordingly, although the most important data are those pertaining to the influence of phase transition on the dispersion relations, such data are very limited.

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